***SOLUTION Section* 3.7 – Power Series**

***Exercise***

(***a***) Find the series’ radius and interval of convergence. For what values of *x* does the series converge (***b***) absolutely, (***c***) conditionally?



***Solution***



When  and  the series diverges.

1. The radius is 1; the interval of converges 
2. The interval of absolute convergence is 
3. There are no values for which the series converges conditionally

***Exercise***

(***a***) Find the series’ radius and interval of convergence. For what values of *x* does the series converge (***b***) absolutely, (***c***) conditionally?



***Solution***



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***Exercise***

(***a***) Find the series’ radius and interval of convergence. For what values of *x* does the series converge (***b***) absolutely, (***c***) conditionally?



***Solution***







When  which is the alternating harmonic series and is conditionally convergent.

 the series diverges harmonic.

1. The radius is ; the interval of converges 
2. The interval of absolute convergence is 
3. The series converges conditionally at 

***Exercise***

(***a***) Find the series’ radius and interval of convergence. For what values of *x* does the series converge (***b***) absolutely, (***c***) conditionally?



***Solution***











When  which is a divergent series

 the series diverges

1. The radius is ; the interval of converges 
2. The interval of absolute convergence is 
3. There are no values for which the series converges conditionally

***Exercise***

(***a***) Find the series’ radius and interval of convergence. For what values of *x* does the series converge (***b***) absolutely, (***c***) conditionally?



***Solution***







1. The radius is ; the series converges for all *x*.
2. The series convergence absolutely for all *x*.
3. There are no values for which the series converges conditionally

***Exercise***

(***a***) Find the series’ radius and interval of convergence. For what values of *x* does the series converge (***b***) absolutely, (***c***) conditionally?



***Solution***







When  which is a convergent conditionally series

 the series diverges

1. The radius is 1; the series converges for .
2. The series convergence absolutely for .
3. The series convergence conditionally for 

***Exercise***

(***a***) Find the series’ radius and interval of convergence. For what values of *x* does the series converge (***b***) absolutely, (***c***) conditionally?



***Solution***







When  which is a divergent series

 the series converges conditionally

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***Exercise***

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***Solution***













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***Exercise***

(***a***) Find the series’ radius and interval of convergence. For what values of *x* does the series converge (***b***) absolutely, (***c***) conditionally?



***Solution***



For the series 







For the series 







When  which is a divergent series

 which is a divergent series

1. The radius is; the series converges for .
2. The series convergence absolutely for .
3. There are no values for which the series convergence conditionally

***Exercise***

Find the radius of convergence of the power series 

***Solution***









By the Ratio Test, the series diverges for  and converges only at its center, 0.

Therefore; the radius of convergence is .

***Exercise***

Find the radius of convergence of the power series 

***Solution***





By the Ratio Test, the series converges for  and diverges for .

Therefore; the radius of convergence is .

***Exercise***

Find the radius of convergence of the power series 

***Solution***









By the Ratio Test, the series converges for all *x*. Therefore; the radius of convergence is .

***Exercise***

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***Solution***









By the Ratio Test, the series diverges for  and converges only at its center, 0.

Therefore; the radius of convergence is .

***Exercise***

Find the interval of convergence of the power series 

***Solution***









So, by the Ratio Test, the radius of convergence is .

The series centered at 0, it converges in the interval 

When 

When 

Therefore; the interval of convergence 

***Exercise***

Find the interval of convergence of the power series 

***Solution***









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***Exercise***

Find the interval of convergence of the power series 

***Solution***









The series converges in the interval 

When 

When 

Therefore; the interval of convergence 

***Exercise***

Find the interval of convergence of the power series 

***Solution***







The series converges for all *x*. Therefore; the interval of convergence 

***Exercise***

Find the interval of convergence of the power series 

***Solution***









The series converges for all *x*. Therefore; the interval of convergence 

***Exercise***

Find the interval of convergence of the power series 

***Solution***







The series converges only for 

***Exercise***

Find the interval of convergence of the power series 

***Solution***









The series converges in the interval 

When 





When 

Therefore; the interval of convergence 

***Exercise***

Find the interval of convergence of the power series 

***Solution***







The series converges in the interval 

When 

When 

Therefore; the interval of convergence 

***Exercise***

Find the interval of convergence of the power series 

***Solution***







The series converges only for 

***Exercise***

Find the interval of convergence of the power series 

***Solution***











The series converges in the interval  and center 

When 

When 

 

Therefore; the interval of convergence 

***Exercise***

Find the interval of convergence of the power series 

***Solution***











The series converges in the interval  and center 

When 





When 



Therefore; the interval of convergence 

***Exercise***

Find the interval of convergence of the power series 

***Solution***











The series converges in the interval  and center 

When 



When 





Therefore; the interval of convergence 

***Exercise***

Find the interval of convergence of the power series 

***Solution***











The series converges in the interval  and center 

When 



When 





Therefore; the interval of convergence 

***Exercise***

Find the interval of convergence of the power series 

***Solution***









The series converges in the interval 

When 

When 

Therefore; the interval of convergence 

***Exercise***

Find the interval of convergence of the power series 

***Solution***









The series converges in the interval 

When 

When 

Therefore; the interval of convergence 

***Exercise***

Find the interval of convergence of the power series 

***Solution***









The series converges in the interval 

When 



When 



Therefore; the interval of convergence 

***Exercise***

Find the interval of convergence of the power series 

***Solution***









Therefore; the interval of convergence 

***Exercise***

Find the interval of convergence of the power series 

***Solution***









Therefore; the interval of convergence 

***Exercise***

Find the interval of convergence of the power series 

***Solution***









Therefore; the interval of convergence 

***Exercise***

Find the interval of convergence of the power series 

***Solution***









The series converges in the interval 

When 

When 

Therefore; the interval of convergence 

***Exercise***

Determine the centre, radius, and interval of convergence of the power series 

***Solution***



The radius of convergence is 1.

The centre of convergence is 0.

The interval of convergence is (−1, 1).

The series does not converge at  or 

***Exercise***

Determine the centre, radius, and interval of convergence of the power series 

***Solution***

 





The radius of convergence is 1, and the centre of convergence is −1. 



Therefore; the given series convergences absolutely on 

At , the series is  which diverges.

At , the series is  which diverges.

Hence, the interval of convergence is .

***Exercise***

Determine the centre, radius, and interval of convergence of the power series 

***Solution***

 





The radius of convergence is 4, and the centre of convergence is 0.

, the given series convergences absolutely on 

At , 

which converges (*p-*series).

At , 

 which also converges.

Hence, the interval of convergence is.



***Exercise***

Determine the centre, radius, and interval of convergence of the power series 

***Solution***

 





The radius of convergence is .

The centre of convergence is 4. 

, which the given series convergences absolutely

At , the series is  which converges (*p-*series).

At , the series is  which also converges (*p-*series).

Hence, the interval of convergence is.

***Exercise***

Determine the centre, radius, and interval of convergence of the power series 

***Solution***

 





The radius of convergence is .

The centre of convergence is 0.

The interval of convergence is the real line 

***Exercise***

Determine the centre, radius, and interval of convergence of the power series 

***Solution***



 





The radius of convergence is .



The centre of convergence is 

The interval of convergence is the real line 

***Exercise***

Determine the centre, radius, and interval of convergence of the power series 

***Solution***



 







The radius of convergence is .

The centre of convergence is  .

The interval of convergence is the real line 

***Exercise***

Determine the centre, radius, and interval of convergence of the power series 

***Solution***



 





The radius of convergence is .

The centre of convergence is  .

The interval of convergence is the real line 

***Exercise***

Determine the centre, radius, and interval of convergence of the power series 

***Solution***



 





The radius of convergence is 

The centre of convergence is 

 

which the given series convergences absolutely

At , the series is  which converges (*p-*series).

At , the series is  which also converges (*p-*series).

The interval of convergence is the real line 

***Exercise***

Determine the centre, radius, and interval of convergence of the power series 

***Solution***



 





The radius of convergence is



The centre of convergence is 

The interval of convergence is the real line 

***Exercise***

Determine the centre, radius, and interval of convergence of the power series 

***Solution***



By Ratio Test:

 





The radius of convergence is 



The centre of convergence is 

 



which the given series convergences absolutely

At , the series is 





which converges *Alternating Harmonic Series*.

At , the series is  which *diverges* (*p-*series )

The interval of convergence is the real line 

***Exercise***

Determine the centre, radius, and interval of convergence of the power series 

***Solution***



By Root Test:

 











The radius of convergence is 

The centre of convergence is 

At , the series is  which diverges by the *divergence Test*.

At , the series is  which diverges by the *divergence Test*.

The interval of convergence is the real line 

***Exercise***

Determine the centre, radius, and interval of convergence of the power series 

***Solution***



By Ratio Test:

 





The radius of convergence is 



The centre of convergence is 

 



which the given series convergences absolutely

At , the series is 





which converges *Alternating Series*.

At , the series is  which *diverges* (*p-*series )

The interval of convergence is the real line 

***Exercise***

Determine the centre, radius, and interval of convergence of the power series 

***Solution***



By Ratio Test:

 









The radius of convergence is 



The centre of convergence is 

 



which the given series convergences absolutely

At , the series is 





which converges *Alternating Series*.

At , the series is 





 *diverges* (*p-*series )

∴ Which *diverges by Comprison Test.*

The interval of convergence is the real line 

***Exercise***

Determine the centre, radius, and interval of convergence of the power series 

***Solution***





By Ratio Test:

 





The radius of convergence is 

The centre of convergence is 



which the given series convergences absolutely

At , the series is 









which diverges *Integral Test*.

At , the series is 









which diverges *Integral Test*.

The interval of convergence is the real line 

***Exercise***

For what value of *x* does the series  converges? What is its sum? What series do you get if you differentiate the given series term by term? For what value of *x* does the new series converge? What is its sum?

***Solution***









When  which is a divergent series

 the series diverges

The series is a geometric series, the sum is



If 



Then 

 is convergent when  and divergent when 

The sum for  is 

***Exercise***

The series  converges to sin*x* for all *x*.

1. Find the first six terms of a series for cos*x*. For what values of *x* should the series converge?
2. By replacing *x* by 2*x* in the series for sin*x*, find a series that converges to sin2*x* for all *x*.
3. Using the result in part (*a*) and series multiplication, calculate the first six term of a series for . Compare your answer with the answer in part (*b*).

***Solution***

1. 





The series converges for all values of *x*.

1. 



1. 







***Exercise***

Find the sum of the series  by the first finding the sum of the power series



***Solution***























Multiply by *x* both sides







Let 





***Exercise***

Find a series representation of  in powers of . What is the interval of convergence of this series?

***Solution***

Let , we have





 

 

 





The radius of convergence of this series is 3.

The distance from the centre of convergence  , to the point −2 where the denominator is 0.

***Exercise***

Determine the Cauchy product of the series . On what interval and to what function does the product series converge?

***Solution***









Let  , then the series holds for 

We have



Then the Cauchy product is







***Exercise***

Determine the power series expansion of  by formally dividing  into 1.

Use the power series 

***Solution***





***Exercise***

Determine the interval of convergence and the sum of the series



***Solution***

 





Therefore; the interval of convergence is 

***Exercise***

Determine the interval of convergence and the sum of the series



***Solution***





 ***Differentiate***

 ***Multiply by x***



Then,







***Exercise***

Determine the interval of convergence and the sum of the series



***Solution***



 



